

Engaging Advanced Math Learners

<http://bit.ly/1n8f8L3>



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Which one doesn't belong?

A

1, 1, 2, 3, 5, 8,...

B

2, 4, 8, 16,...

C

-1, 1, 3, 5, 7,...

D

100, 99, 98, 97,...

Indicators of Mathematical Giftedness

- * Unusual curiosity about numbers and mathematical information
- * Ability to understand and apply ideas quickly
- * High ability to see patterns and think abstractly
- * Use of flexible and creative strategies and solutions
- * Ability to transfer a mathematical concept to an unfamiliar situation
- * Use of analytical, deductive, and inductive reasoning
- * Persistence in solving difficult and complex problems

Mathematical giftedness can manifest in three ways:

- * Analytical. Analytically gifted maths students tend to think abstractedly with ease. They solve problems using logic and reasoning.
- * Geometric. Geometrically gifted maths students prefer using diagrams and visual aids to solve problems.
- * Harmonic. Harmonically gifted maths students can use both geometric and analytic methods of thinking with ease.

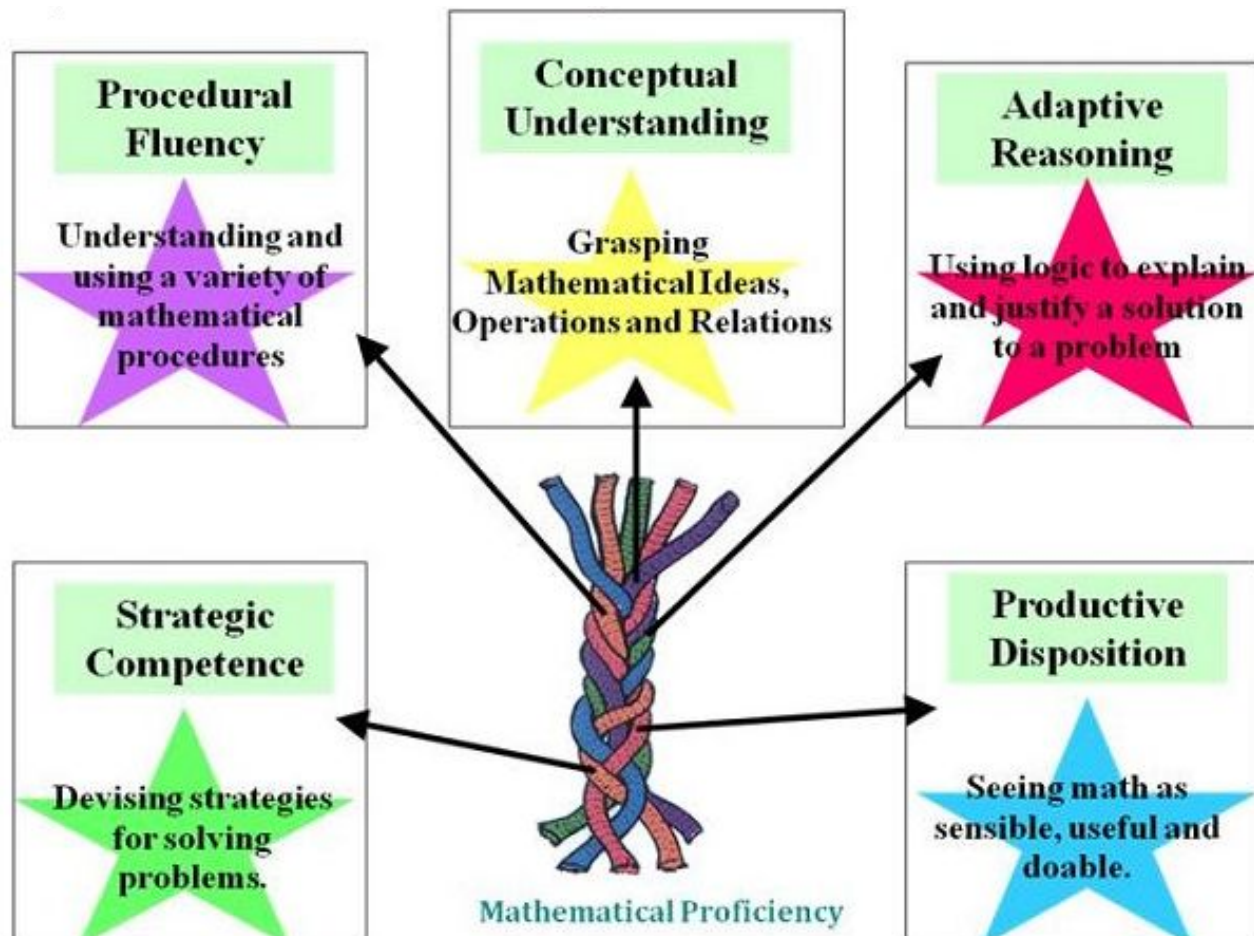
Common Core Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make sense of structure.
8. Look for and express regularity in repeated reasoning.

Effective Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem-solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

(Principles in Action , 2014, National Council of Teachers of Mathematics)



Conceptual Understanding

- * High ability to see patterns and think abstractly
- * Use of analytical, deductive, and inductive reasoning

Teachers:

EP1. Establish mathematics goals to focus learning.

EP2. Implement tasks that promote reasoning and problem-solving.

To promote students' ability to:

MP2. Reason abstractly and quantitatively.

MP7. Look for and make sense of structure.

Productive Disposition

* Ability to transfer a mathematical concept to an unfamiliar situation

Teachers:

EP3. Use and connect mathematical representations.

To promote students' ability to:

MP4. Model with mathematics.

MP5. Use appropriate tools strategically.

Adaptive Reasoning

* Unusual curiosity about numbers and mathematical information

Teachers:

EP4. Facilitate meaningful mathematical discourse.

EP8. Elicit and use evidence of student thinking.

To promote students' ability to:

MP3. Construct viable arguments and critique the reasoning of others.

Strategic Competence

- * Persistence in solving difficult and complex problems
- * Use of flexible and creative strategies and solutions

Teachers:

EP5. Pose purposeful questions.

EP7. Support productive struggle in learning mathematics.

To promote students' ability to:

MP1. Make sense of problems and persevere in solving them.

Procedural Fluency

* Ability to understand and apply ideas quickly

Teachers:

EP6. Build procedural fluency from conceptual understanding.

To promote students' ability to:

MP6. Attend to precision.

MP8. Look for and express regularity in repeated reasoning.

Conceptual Understanding

Activities that develop a deep conceptual level of understanding

“Rich” tasks

Higher level on Bloom’s taxonomy

Multiple solutions or paths to solution

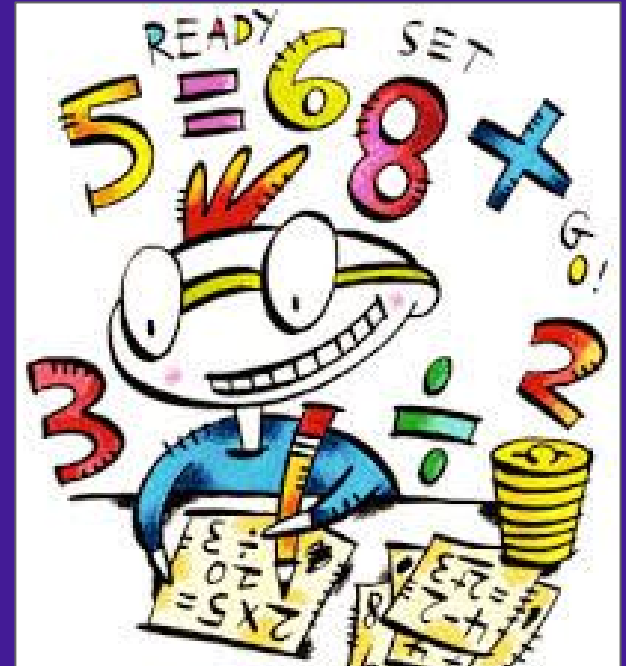
Non-algorithmic problem-solving

Open-ended questions

Constructivist

Exploratory

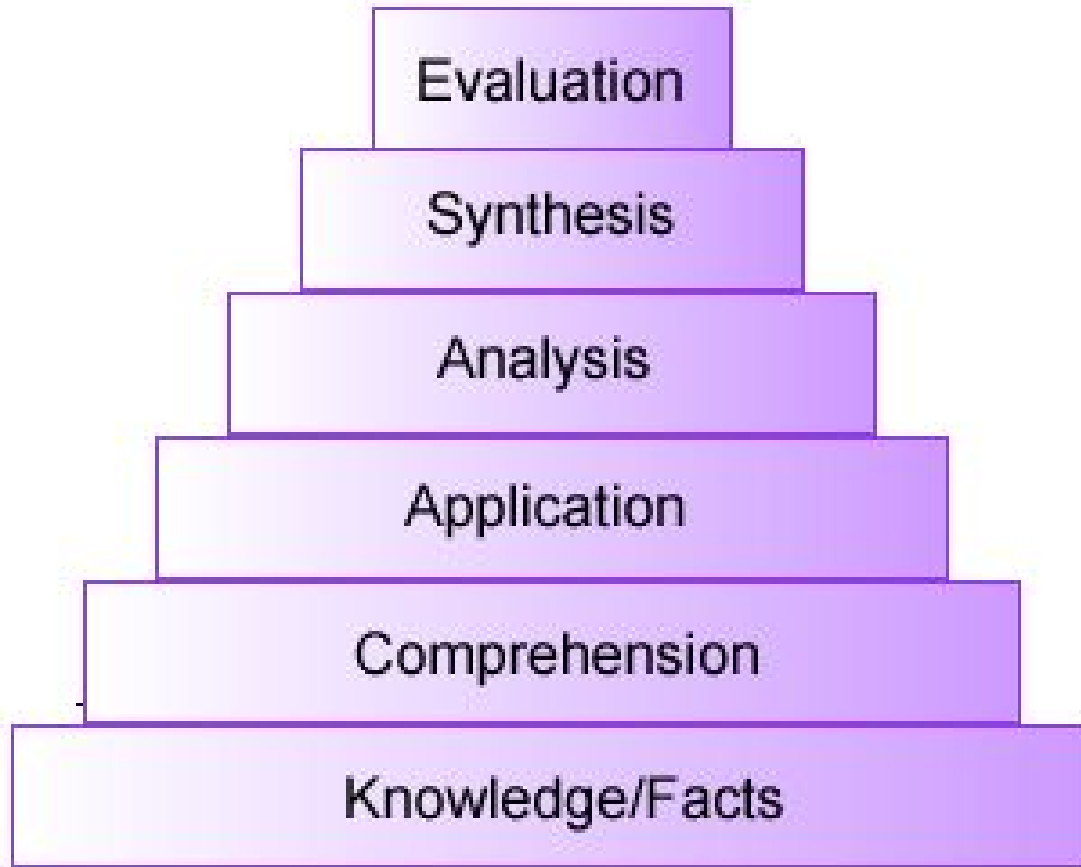
CGI (cognitively-guided instruction)



Rich tasks (good problems):

- * are accessible to a wide range of learners
- * challenge learners to think for themselves
- * offer different levels of challenge, different methods, and different responses
- * have the potential to broaden students' skills and/or deepen and broaden mathematical content knowledge
- * encourage creativity and imaginative application of knowledge
- * have the potential for revealing patterns or underlying principles
- * encourage collaboration and discussion
- * encourage learners to develop confidence and independence as well as to become critical thinkers

Concrete ----- Abstract
Simple ----- Complex



Non-algorithmic Problem-solving

When writing or adapting problems, consider:

Familiar context

Action

Personalize

Multiple paths to solution

May be more than one acceptable answer

Discourse of students' strategies

Your turn! Pick one of the grade level standards on the following slide and come up with at least three ideas/questions that could be used to add depth and/or complexity to a task. You may work in a group. Please be ready to share some ideas after your discussion so that others can benefit from your expertise.

All of the standards deal with **area**, but there is a clear progression across grade levels.

* Suggested scenario: building a clubhouse, a dog pen, etc.

Third grade: 3.MD7.B

Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Fourth grade: 4.NBT.5

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Fifth grade: 5.NF.4

Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Third grade ideas

Fourth grade ideas

Fifth grade ideas

- **Math**
 - Addition and subtraction of fractions
 - Multiplication and division of whole numbers
 - Geometry: area, perimeter, volume
 - Measurement: length, weight, capacity
- **Science**
 - Earth and space science
 - Life science
 - Physical science
- **Language Arts**
 - Reading comprehension
 - Writing: narrative, expository, persuasive
 - Grammar and mechanics
- **History and Social Studies**
 - American history
 - World history
 - Civics and government
- **Art and Music**
 - Visual arts
 - Music theory and performance
- **Physical Education**
 - Health and fitness
 - Sports and games

Adaptive Reasoning

Activities that challenge students, allow them to “play with numbers,” and pique their interest

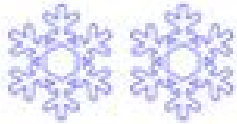
Which one doesn't belong?

Question of the day

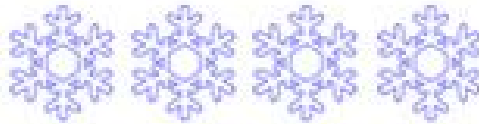
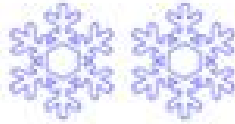
Visual Patterns

Greg Tang Math

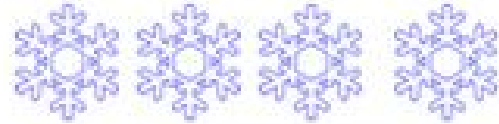
MathPickle



step 1



step 2



step 3

How many snowflakes would be in step 43? What expression would describe this pattern, where s is the number of steps?

$$3s + 3$$

Adaptive Reasoning

Make sure that students have time at the end of class to share strategies and solutions.

Pick 3 students to share: low, middle, high.

Encourage cooperative group work.

Set rules for discourse: respect, agree/disagree, connections.

When other students share their thinking, I will:

- * Look for **patterns** in the problems or answers.
- * Be ready to **share strategies** and **explain** my thinking.
- * **Listen** to other mathematicians' thinking and be ready to **respond**:

I agree with _____ because _____.

I don't understand _____. Can you explain it again?

I disagree with _____ because _____.

How did you decide to _____?

Strategic Competence

Be alert for ways to extend learning for those who are ready

Questioning strategies (HOTS)

Using questioning to stimulate mathematical thinking

Questions for math class (elementary)

How to write higher order math questions

What questions can we ask when students are struggling?

What information do you have?

What does the question ask? (What do you have to find out?)

Where could you start?

What have you already tried?

What do you think you should do?

Are there other ways you could try to solve it?

Can you draw a picture to show the information?

How did you get that answer?

Procedural Fluency

Activities that develop number sense and place value concepts

Encourage estimation so students learn to evaluate whether or not their answers are reasonable

Number talks

Visual fraction representations

“Reasoning” problems

Fluency: Accuracy, efficiency, flexibility

BC

HOW MUCH IS 43.789 TIMES 5.221 ?

228.622.369.

NOT ONLY ARE RABBITS CUTE. THEY HAVE THE ABILITY TO MULTIPLY AT AMAZING SPEED.

3rd/4th grade

$$93 - 28$$

$$76 - 39$$

$$236 - 188$$

$$856 - 687$$

4th grade

$$3 \times 7$$

$$7 \times 10$$

$$13 \times 17$$

$$17 \times 14$$

$$14 \times 16$$

4th grade

$$4 \times 8$$

$$8 \times 8$$

$$16 \times 4$$

$$32 \times 2$$

$$64 \times 1$$

5th grade

$$18 \times 3.5$$

$$4.5 \times 24$$

$$12 \times 9.5$$

$$9.5 \times 1.2$$

$$.95 \times 1.2$$

4th/5th grade

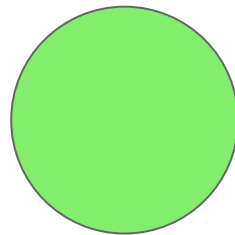
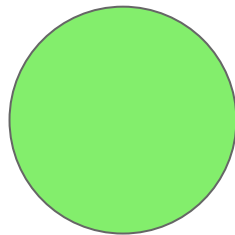
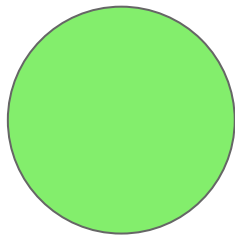
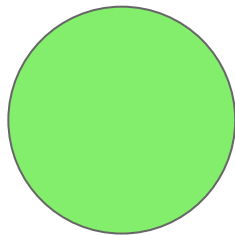
$$300 \div 6$$

$$600 \div 6$$

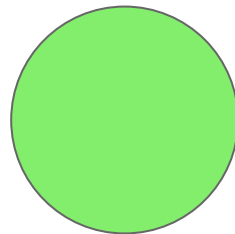
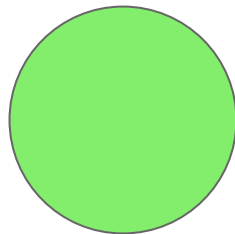
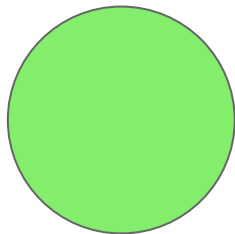
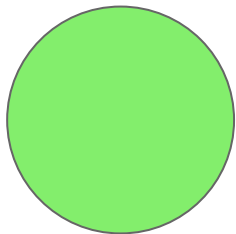
$$600 \div 12$$

$$1200 \div 24$$

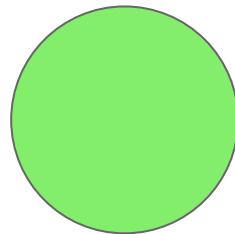
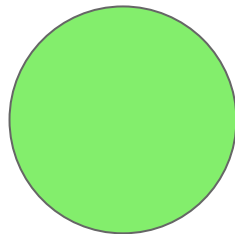
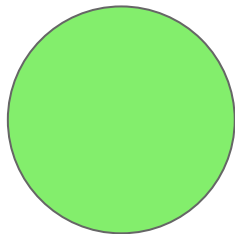
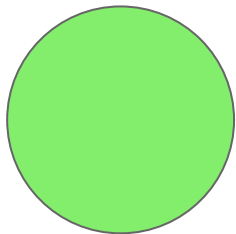
$$600 \div 24$$



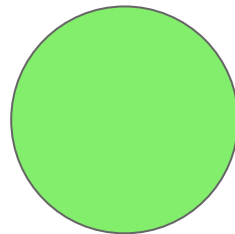
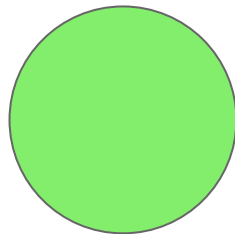
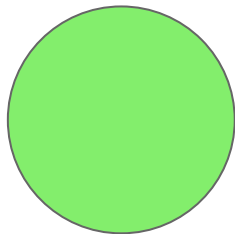
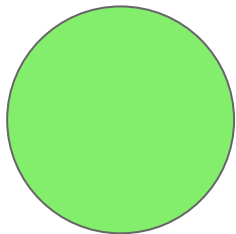
$$\frac{1}{2}$$



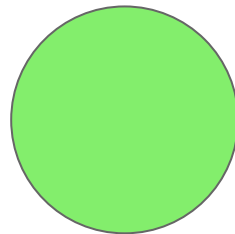
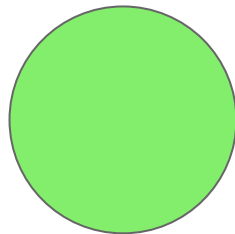
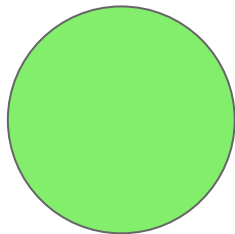
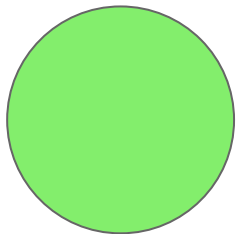
$$\frac{1}{4}$$



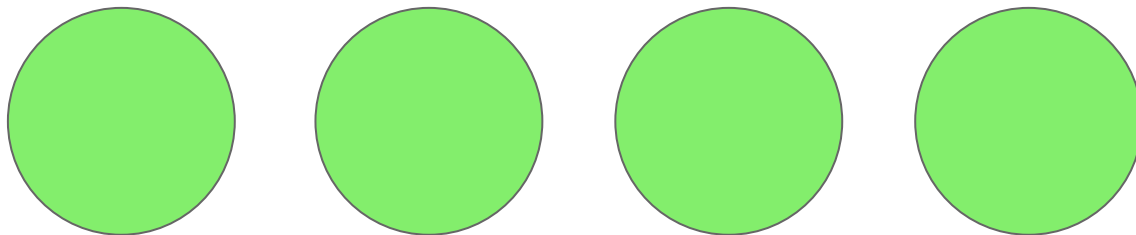
$$\frac{1}{3}$$



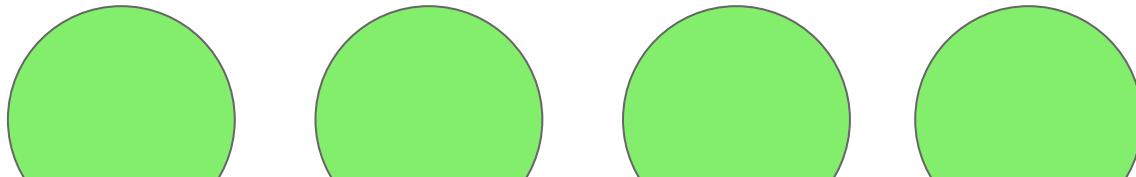
$$\frac{1}{6}$$



$$\frac{1}{8}$$

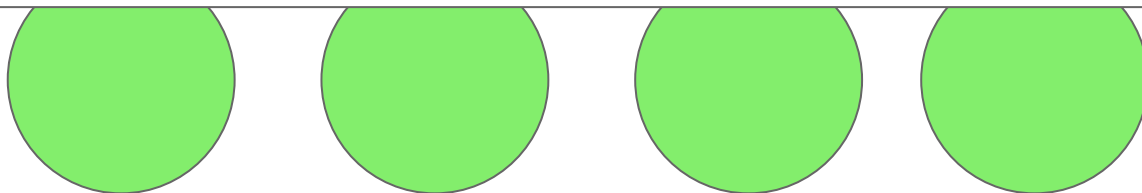


$$\frac{1}{2}$$

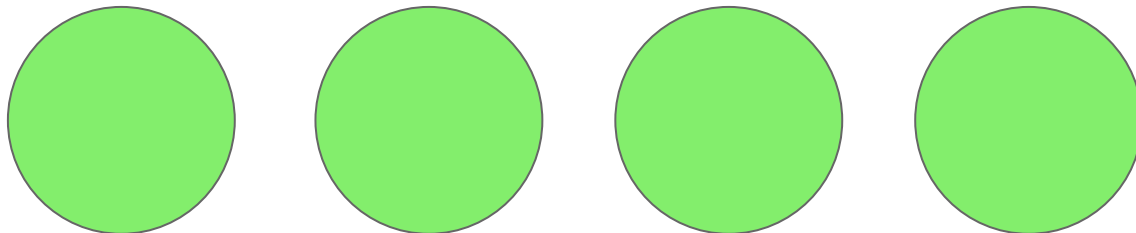


$$\frac{1}{4}$$

What understandings about fractions are you promoting in students?



$$\frac{1}{8}$$



$$\frac{1}{3}$$

$$\frac{1}{6}$$

Joey multiplied three numbers together and got 24. What three numbers could he have multiplied?

What strategy did you use to figure out the numbers?

(from hcpss.org)

MCC4.NBT.6

Which has the greater quotient:

$2686 \div 6$ or $3612 \div 7$?

Without solving the problem,
explain which quotient is greater
and why.

You have the decimals 6.29, 5.85, 5.69, 6.18, and 5.72. Which decimal is closest to 6? How do you know? Be prepared to defend your answer.

You have the decimals 7.57, 7.75, 7.69, 7.48, and 7.52. Which decimal is closest to $7\frac{3}{5}$? How do you know? Be prepared to defend your answer.

Determine if these equations are true or false. Defend your answer using your knowledge of place value and the commutative, associative, and/or distributive properties.

a. $6 \text{ tens} = 2 \text{ tens} \times 3 \text{ tens}$

b. $44 \times 20 \times 10 = 440 \times 2$

c. $86 \text{ ones} \times 90 \text{ hundreds} = 86 \text{ ones} \times 900 \text{ tens}$

d. $64 \times 8 \times 100 = 640 \times 8 \times 10$

e. $57 \times 2 \times 10 \times 10 \times 10 = 570 \times 2 \times 10$

(from engageNY.org)

Compare these fractions:

$$3/7, \frac{2}{5}, 5/9$$

Which fraction is closest to $\frac{1}{2}$? How do you know?

Resources-Books

Carpenter, T.P., Fennema, E., Franke, M.L., Levi, L., & Empson, S.B. (2014). *Children's mathematics: Cognitively guided instruction* (2nd ed.). Portsmouth, NH: Heinemann.

Empson, S.B., & Levi, L. (2011). *Extending children's mathematics: Fractions and decimals*. Portsmouth, NH: Heinemann.

Schuster, L., & Anderson, N.C. (2005). *Good questions for math teaching: Why ask them and what to ask, grades 5-8*. Sausalito, CA: Math Solutions.

Sullivan, P., & Lilburn, P. (2005). *Good questions for math teaching: Why ask them and what to ask, grades k-2*. Sausalito, CA: Math Solutions.

van de Walle, J.A. (2007). *Elementary and middle school mathematics: Teaching developmentally*. Boston: Pearson.

van de Walle, J.A., & Lovin, L.H. (2006). *Teaching student-centered mathematics, grades 4-6*. Boston: Pearson.

Resources-Websites

[Common Core Progressions](#)

[Mathwire problem-solving](#)

[Delaware City Schools 5th Grade Enriched Math](#) (grades 3 and 4 also accessible from this page)

[Howard County Public School System](#)

[Nrich Math](#)

[Illuminations](#)

[OER Commons](#)

[Inside Mathematics](#)

[Teacher Created Common Core Math Tasks](#) (grade level tasks)

[Georgia Standards](#)

[RDA Performance Task Bank](#)

[Read Tennessee Math K-3](#)

[Cristo Rey Middle Level Mathematics Routine Bank](#)

[Engage NY](#)

[Florida Go Math](#)

[Math Learning Center Free Apps](#)

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presentation!

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information.